Physics 114 – Test 2

Thursday, March 6, 2003

Geometric factors: You may find some of the following formulas helpful:

Sphere: Volume $V = \frac{4}{3}\pi r^3$ Area $A = 4\pi r^2$

Circle: Area $A = \pi r^2$ Circumference $C = 2\pi r$

Part One – Do any four of the following five problems (10 points each)

1. For each of the following combination of units, give a single name for the resulting unit. For example, if I gave you A·Ω, the answer would be V.

a) T·m²

Magnetic field multiplied by area yields magnetic flux, which is measured in Webers, so $\mathrm{T{\cdot}m^2} = \mathrm{Wb}$.

b) $\dfrac{1}{\sqrt{\mathrm{F \cdot H}}}$

Comparing this to the formula $\omega = \dfrac{1}{\sqrt{LC}}$, we see that L has the units of inductance (Henrys, H) and C has the units of capacitance (Farads, F), so the resulting combination must have the unit of angular frequency ω, which is Hertz, Hz.

Alternatively, simply substitute the definitions. H = V·s/A, and F = C/V, so we have

$$\frac{1}{\sqrt{\mathrm{F \cdot H}}} = \frac{1}{\sqrt{(\mathrm{C/V})(\mathrm{V \cdot s/A})}} = \frac{1}{\sqrt{\mathrm{C \cdot s/A}}} = \frac{1}{\sqrt{\mathrm{C \cdot s/(C/s)}}} = \frac{1}{\sqrt{\mathrm{s}^2}} = \frac{1}{\mathrm{s}} = \mathrm{Hz}$$

c) $\mathbf{s^{-1}}$

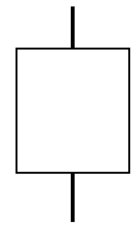
An inverse second is a Hertz, Hz.

d) H·Hz

A Henry is V·s/A, and Hz is an inverse second, so H·Hz = V/A = Ω. Alternatively, note that inductive reactance is given by $X_L = L\omega$, and since L has units of H and ω has units of Hz, the product must have the units of inductive reactance, which is ohms Ω.

e) $\dfrac{\mathrm{N}}{\mathrm{A \cdot m}}$

This is one of the ways of defining a Tesla. Alternatively, note that A·m = C·m/s, so if you look at formulas like $F = qvB$, we can see that magnetic field has units like F/qv, which is like N/(C·m/s). So this is a Tesla T.



2. A mystery box has two leads coming out of it. This mystery box contains a resistor, a capacitor, or an inductor. In each of the three cases, explain how one could determine what is in the box, and how one would measure the resistance, capacitance, or inductance of the object. For example, one-third of your answer might say, "only a resistor allows direct current to pass through, so you can use a battery, and deduce the resistance using the formula $R = \Delta V \cos I$." Give any relevant formulas.

The method you describe may use any type of electrical instruments, and you can measure the voltage across the mystery box, or the current through the box, but you may not open the box or measure anything interior. For example, you cannot say "I would measure if the component is heating up, since that would tell me it is a resistor."

There are several methods that would work. One way to do this is to attach an AC source to the mystery box, and measure the voltage and current as a function of time. If the voltage and the current are synchronized, then there is a resistor. If the current lags the voltage (rises and peaks later), then it is an inductor. If the current leads the voltage (rises and peaks earlier), then it is a capacitor. The frequency can presumably controlled, or if not, it can be measured by examining the graph of voltage vs. time (see problem 6). Then the current is related to the voltage by one of the following three formulas:

$$\Delta V = IR$$

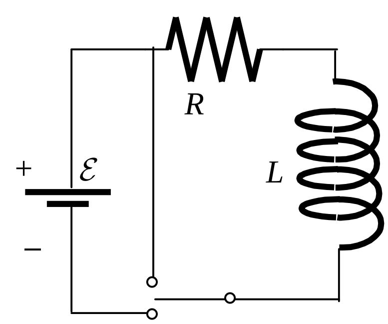
$$\Delta V = IX_L = IL\omega$$

$$\Delta V = IX_C = \frac{I}{C\omega}$$

These formulas can be used with either maximum values or rms values; they work in either case.

If measuring the phase shift is difficult, it is also possible to vary the frequency , say increasing it, and see whether the current rises (capacitor), falls (inductor), or stays the same (resistor). Many other strategies would work as well.

3. Consider the circuit at right, containing a battery, a resistor, an inductor, and a two-way switch



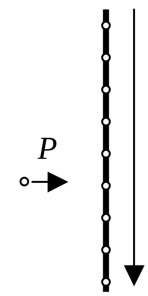
a) The switch is moved to the bottom position, and left there for a long time. Describe qualitatively the long-term behavior of the circuit, explaining where current is flowing, and giving any relevant formulas.

Eventually the system will reach a steady state, which means that the current will be constant. With the switch in the down position, the current will flow out of the battery, through the resistor, then through the inductor, and back to the battery. The inductor provides no resistance in the steady state, so the current will flow through the resistor according to Ohm's Law, $\Delta V = IR$, so $I = \mathcal{E}/R$.

b) **At t = 0, the switch is suddenly moved to the upper position, bypassing the battery. Describe qualitatively the behavior at later times, and give any formulas that you think are relevant.**

Realistically, switches are not instantaneous, but we have to treat the problem as if it is instantaneous. The inductor, which had a steady current in it, resists sudden changes in current. Therefore, current will continue to flow through the inductor. The only way it can flow is through the switch and the resistor, continuing to circulate in a clockwise direction. However, the circuit is now an LR circuit, and the current will steadily die away as the energy in the inductor is converted to heat in the resistor. The current dies away with the characteristic time $\tau = L/R$.

4. A wire contains negatively charged electrons, which are flowing from top to bottom.



a) **Which direction is the current flowing?**

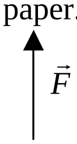
Current is flowing the <u>opposite</u> of the direction that the negatively charged electrons are flowing, and therefore, the current is flowing upwards.

b) **Which direction is the magnetic field pointing at some of the points near the wire? In particular, which direction is the magnetic field at the point *P*?**

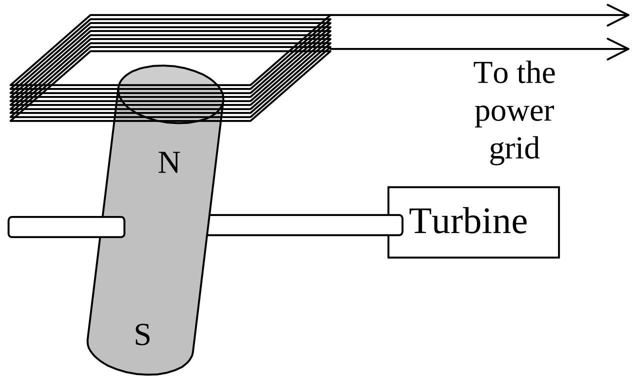
Placing our thumb in the direction the current is flowing (up), we see that our fingers curl around the wire in such a way that they point out of the paper to the left of the wire, and into the plane of the paper on the right of the wire. In particular, the magnetic field is pointing out of the plane of the paper at the point P.

c) **At the point *P* there is another negatively charged electron which is moving directly towards the wire. What direction is the magnetic force on the electron?**

The magnetic force on a particle is given by $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$. Placing our fingers in the direction of the motion (to the right), we find that when we curl our fingers, they will point out of the plane of the paper if we place the back of our hand against the paper. This only happens if our thumb is pointing downwards. Hence this is the direction of the cross-product. <u>However</u>, the charge is again negative, so the force gets reversed, and the force is actually towards the top of the paper.



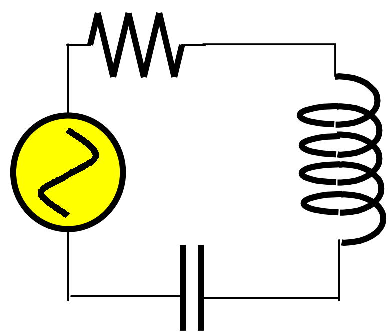
5. Consider the proposed AC generator sketched at right. It consists of a permanent magnet which is rotated rapidly, so that first the north pole of the magnet and then the south pole are near the loop. The loop has several turns of wire. Would this AC generator work? Give any relevant formulas.



Yes, it would work, although as described, it might not yield a nice, neat, sinusoidal AC signal. Faraday's law of induction applies whether the magnetic field changes due to motion of the coil or motion of the magnet (as demonstrated in class). The EMF generated by a single loop is simply given by $\mathcal{E} = -\frac{d\Phi_B}{dt}$, where Φ_B is the magnetic flux through the loop at time t. As the magnet is rotated back and forth, the flux would change sign, and the voltage generated in the loop would correspondingly change sign as well. Because there are several loops, we must then multiply this by N, the number of loops.

PART TWO – Do any two of the following three problems (30 points each)

6. The circuit shown at right has a variable AC source, a resistor, an inductor, and a capacitor. The AC source is measured to produce a voltage as sketched in the graph below (the solid curve). The current is measured as well, and also shown in the graph below.



a) What is the period T, frequency f, and angular frequency ω that the AC source is operating at?

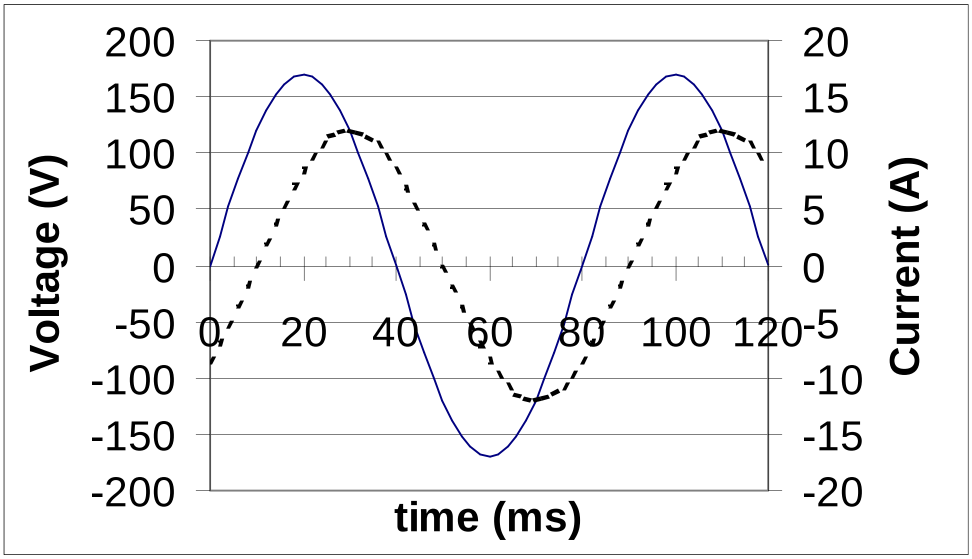
Looking at the graph, we see that the peaks of the voltage are about 80 ms apart, so the period is T = 80 ms = 0.08 s. The frequency is just the reciprocal of this, so $f = 1/T$ = 12.5 Hz. The angular frequency can be found from $\omega = 2\pi f = 78.54$ Hz .

b) What is the maximum voltage ΔV_{max} and the maximum current I_{max} for this circuit? What is the root-mean-square voltage and current ΔV_{rms} and I_{rms}?

The maximum voltage is the highest value the voltage attains over a cycle. Looking at the graph, this is between 150 V and 2000 V, and a little closer to 150 V, so I estimate that it is 170 V. Similarly, the current maxes out between 10 and 15 A, and a little closer to 10, so I estimate it at 12 A.

The rms voltage and current are simply obtained by dividing these by $\sqrt{2}$. The answers are:

ΔV_{max} = 170. V, $\quad I_{max}$ = 12.0 A, $\quad \Delta V_{rms}$ =120. V, $\quad I_{rms}$ = 8.5 A.



Voltage (solid curve, left scale) and current (dotted curve, right scale) as a function of time for problem 6. The time is in milliseconds, and the voltage and current are in volts and amps respectively.

c) What is the impedance Z for this circuit?
The impedance can be obtained from

$$Z = \frac{\Delta V_{\max}}{I_{\max}} = \frac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = 14.2\ \Omega$$

d) **Does the current lag (come after) or lead (come before) the voltage? Is the phase difference ϕ positive or negative? Estimate the phase difference ϕ. You may use radians or degrees.**

The current rises and peaks after the voltage does. Therefore, the current lags the voltage. You can get the phase shift most easily by looking at the place where the voltage and current cross the time axis. The voltage crosses zero precisely at $t = 0$, while the current crosses at approximately $t = 10$ ms. Since the period is 80 ms, this is one-eighth of a cycle. Dividing a full cycle, which corresponds to 2π radians or 360°, by 8, the phase shift is $\frac{1}{4}\pi$ radians, or 45°.

e) Which is bigger, the inductive reactance X_L or the capactitative reactance X_C? If we wished to change the frequency of the source so that the frequency was at the resonance frequency of the circuit, should we increase *f*, decrease *f*, or is it already tuned correctly?

We use the formula $\tan\phi = \dfrac{X_L - X_C}{R}$ to deduce that

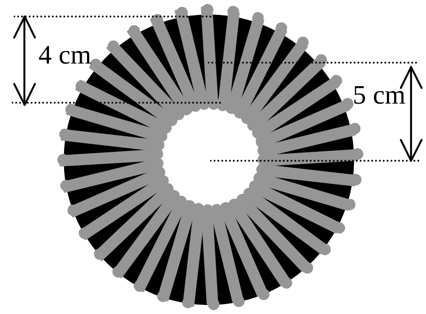
$$\frac{X_L - X_C}{R} = \tan\phi = \tan 45^{\circ} = 1,$$

so we conclude that $X_L - X_C = R$. Since the resistance R is positive, we must have $X_L > X_C$.

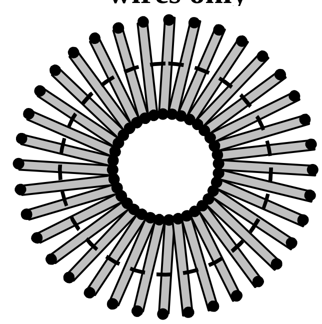
To get to resonance, we have to make the inductive reactance match the capacitive reactance, by lowering the former, or raising the latter, or both. Since $X_L = L\omega$, and $X_c = \dfrac{1}{C\omega}$, this can be accomplished by lowering the angular frequency ω, which is the same as lowering the frequency f.

7. A tokamak is a donut shaped electromagnet, sort of like a solenoid bent into a circle. It is typically formed by wrapping wires repeatedly around a hollow donut-shaped shell. The solenoid shown below has a major axis of 5 cm (that means that the distance from the center of the "hole" to the center of the turns is 5 cm) and a minor axis of 2 cm (that means that the distance across the "cake" part of the donut is 4 cm). The donut has 36 windings each carrying 150 A of current. The current is flowing in such a way that the current comes up (out of the plane of the page) in the "hole" of the donut, outwards across the top of the donut, back down (into the plane of the paper) around the outside edge of the donut, and back inwards on the bottom of the donut.

Outside view: shell and wires



Cutaway view: wires only



a) **What is the total current, and direction of current flowing through a circle of radius *r* = 5 cm (the dashed circle)? Is the answer any different for *r* = 4 cm or *r* = 6 cm?**

The current is flowing out of the plane of the page in the center part, and this is inside our dashed circle. Each of the wires carries 150 A, and since there are 36 of them, the total current flowing out of the plane of the paper is 5400 A. The answer is the same for r = 4 cm and r = 6 cm, because the wires in the center still pass through this circle, and the wires on the outside do not.

b) **What is the direction of the magnetic field on the dashed circle? Use Ampere's Law to calculate the magnitude of that magnetic field. What, if anything, changes if you calculate the magnetic field at *r* = 4 cm or *r* = 6 cm?**

A solenoid has a magnetic field that points along its axis. If you look, say, at the top of the donut, the current will flow in the direction of your curled fingers if you point your thumb to the left. By a similar argument, it will point in a counter-clockwise direction all around the dashed curve. So the magnetic field points counter-clockwise everywhere.

As an alternative argument, there is a net current flowing upwards in the inner part of the circle. Pointing your thumb upwards, you see that your fingers curl counter-clockwise, so again, the field is pointed counter-clockwise around the circle.

Now, what is the magnitude of this magnetic field? We can calculate this easily using Ampere's Law. Since we don't have any changing electric fluxes, we can use the simple form

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$. Since the magnetic field is pointed along the direction of the circle, the integral is just the magnetic field times the circumference, or $2\pi rB$. So we have

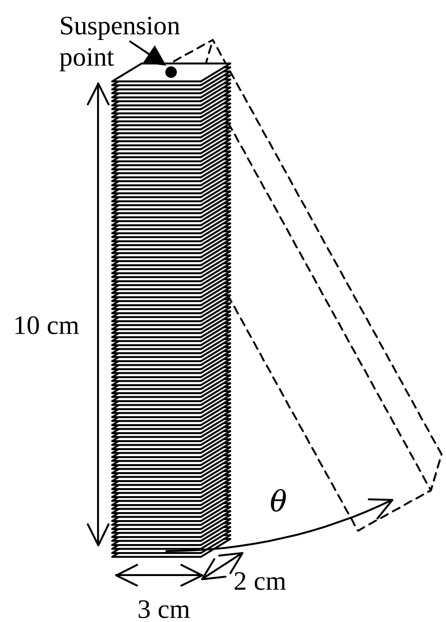
$$2\pi rB = \mu_0 I \text{ , so that } B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(5400\text{ A}\right)}{2\pi\left(0.05\text{ m}\right)} = 0.0216\text{ T} = 21.6\text{ mT}$$

The only thing that changes if you calculate the flux at r = 4 cm or r = 6 cm is in the final step, when you divide by r. The magnetic field at these distances are 27 mT and 18 mT respectively.

c) How much current is there flowing through a circle of radius 8 cm (outside the entire donut)? How about radius 2 cm (in the hole of the donut)? What is the magnetic field there?

A circle of radius r = 2 cm would be entirely inside the hole of the donut, so there would be no current penetrating it. Ampere's Law can still be used, but the total current is 0, so there is no magnetic field. At r = 8 cm, the circle would be bigger than the entire Tokamak. There are 36 wires carrying 150 A each coming up in the center of the donut, but also 36 wires carrying 150 A going back down around the outer perimeter of the donut, for a total current of 0A. So in both cases, the magnetic field (in the hole and outside the donut) is zero.

8. A pendulum is formed as follows. A solenoid is formed with a cross-section that is a rectangle (size 2 cm × 3 cm) with 100 windings/cm and a length of 10 cm and mass 100 g. A current of 2 A is then turned on, in such a way that when the solenoid hangs down, the current is running counter-clockwise around the solenoid. It is then dangled vertically from one end in a gravitational field g = 9.8 m/s^2. A vertical magnetic field of magnitude B = 1 T is turned on in the +z direction.



a) If the solenoid is hanging at an angle θ compared to the vertical, what is the magnetic flux through one turn of the solenoid? What is the total flux through all the turns of the solenoid?

The magnetic flux is equal to the magnetic field dotted into the area. The solenoid has a cross-sectional area of $A = ab = \left(0.02\text{ m}\right)\left(0.03\text{ m}\right) = 0.0006\text{ m}^2$. The angle θ is the same as the angle between the vertical direction (the way the magnetic field is pointing) and the direction of the normal to a single loop, so the flux is

$$\Phi_B = \vec{A}\cdot\vec{B} = AB\cos\theta = \left(0.0006\text{ Wb}\right)\cos\theta$$

Since there are 100 windings per cm, and the solenoid is 10 cm long, there are a total of 1000 windings, so we simply multiply this by 1000 to get the total for the whole solenoid:

$$\Phi_B = N\left(\vec{A}\cdot\vec{B}\right) = AB\cos\theta = \left(0.60\text{ Wb}\right)\cos\theta$$

Some people interpreted the question differently, thinking I was asking about the magnetic field caused by the solenoid through itself. This can be computed by the formula $B = \mu_0 NI / L = 25\text{ mT}$ to get the magnetic field, then multiply by the area to get 1.5×10^{-5} Wb through a single coil, or 0.015 Wb through all 1000 coils. However, in this case, there is no factor of $\cos\theta$, since the solenoid is obviously aligned with itself. Technically, this factor can be added to the previous one to give the total flux.

b) **What is the magnetic potential energy of the solenoid due to its interaction with the magnetic field, if it is hanging straight downwards, and if it is hanging straight upwards?**

For a loop of wire with a current *I*, the magnetic potential energy is $U = -I\vec{A}\cdot\vec{B}$. Again, you have to multiply by the number of coils, $U = -IN\vec{A}\cdot\vec{B}$. This is just the current *I* times the previous answer, so we have

$$U = -\Phi_B I = -(1.20\text{ J})\cos\theta$$

But wait! We have to make sure we get the sign right! This sign is correct provided we have calculated flux in accordance with the right hand rule. As viewed from the top, the current is flowing around in a counter'clockwise manner, and if you curl your fingers in such a manor, you will find your thumb pointing upwards. So, indeed, we have the sign right, and the energy is calculated correctly. Now, for it hanging up or down, we simply plug in $\theta = 0$ or $\theta = 180^\circ$, and find a total energy of –1.20 J when it is hanging down, and +1.20 J when it is pointing up.

Again, some people attempted to calculate the energy caused by the solenoid's interaction with its own magnetic field. This is given by $U = \frac{1}{2}LI^2$. We can get the inductance *L* from the formula $L = \dfrac{\mu_0 N^2 A}{\ell} = 7.54\text{ mH}$, so with 2 A running through the wire we have $U = 0.015$ J. Again, it does not depend on the angle.

c) The background magnetic field is adjusted so that the solenoid is indifferent; it prefers neither to hang downwards nor upwards. Which direction must the magnetic field be, and how large must it be? The gravitational potential energy is given by *U* = *mgh*, where *h* is the height of the center of mass of the solenoid.

The magnetic field, as it is arranged, has its energy at a minimum when the solenoid points down, so it prefers, magnetically, to hang down. Gravity, of course, also prefers things to hang down, so clearly the two forces do not oppose each other.

If we change the magnetic field from 1 T to *B*, then the magnetic potential energy will alter to $U_B = -(2.40\text{ J})(B/\text{T})\cos\theta$. The gravitational potential is given by *mgh*, where *h* is the center of mass. The distance from the top of the solenoid to the midpoint is 5 cm. When it is hanging downwards, this represents a negative height, and when it is upwards, it is a positive height, so $h = -(0.05\text{ m})\cos\theta$. The gravitational potential energy is therefore

$$U_g = mgh = (0.1\text{ kg})(9.8\text{ m/s}^2)(-0.05\text{ m})\cos\theta = -(0.049\text{ J})\cos\theta$$

The total energy is the sum of these two, which is

$$U = U_g + U_B = -\left[(0.049\text{ J}) + (2.40\text{ J})\left(\frac{B}{\text{T}}\right)\right]\cos\theta$$

This is zero, which means the solenoid doesn't care which way it points, if the argument in square brackets vanishes, which happens when

$$(0.049 \text{ J}) + (2.40 \text{ J})\left(\frac{B}{\text{T}}\right) = 0, \qquad \text{or} \qquad B = -0.0204 \text{ T}$$

The minus sign in this formula just signals that the magnetic field must be reversed, so that it points downwards.

What about the self-energy of the inductor? This is 0.015 J no matter which direction the solenoid hangs. Therefore, this contribution is irrelevant to calculating the background field required.